Povzetek

V diplomski nalogi je obravnavana homogenizacija heterogenega materiala po metodi lastnih deformacij in lastnih napetosti, podrobneje homogenizacija materiala s periodično mikrostrukturo po metodi Nemat-Nasserja [16] z uporabo Fourierovih vrst. Obravnavamo linearno elastične materiale in majhne deformacije. Efektivne konstitutivne lastnosti določimo po teoriji povprečnih polj na reprezentativnem volumskem elementu (RVE). S postopkom homogenizacije RVE nadomestimo z ekvivalentno homogeno snovjo enake oblike in velikosti, ki ima na območju vključkov definirano polje lastnih deformacij ali lastnih napetosti, s čimer dosežemo isti odziv originalnega heterogenega RVE in njegove ekvivalentne homogene različice. Vrednost lastnih deformacij ali lastnih napetosti ocenimo po Willisovem postopku [22]. Če je mikrostruktura periodična, so tudi polja pomikov, deformacij in napetosti periodična, zato jih razvijemo v Fourierovo vrsto. S tem dobimo enostavno in močno metodo aproksimacije efektivnih konstitutivnih lastnosti heterogene snovi s periodično mikrostrukturo, ki jo testiramo na okroglih in elipsoidnih vključkih oblike diska in rezultate primerjamo z rezultati homogenizacije po metodi Cohena [2] in Kushcha [12]. Na območju vključka sta po Nemat-Nasserjevi aproksimativni metodi dejansko deformacijsko polje in lastno deformacijsko polje povezana s konstantnim tenzorjem, za katerega pokažemo, da konvergira k pripadajočemu Eshelbyjevemu tenzorju, ko se razmak med vključki povečuje.

Math. Subj. Class. (MSC 2000): 74Q05 74Q15

Ključne besede:
mikromehanika, homogenizacija, periodična mikrostruktura, Eshelbyjev tenzor

Keywords:
micromechanics, homogenisation, periodic microstructure, Eshelby tensor

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