Povzetek

Injektivno barvanje grafov je barvanje, ki dvema vozliščema priredi različno barvo, če imata skupnega soseda, to je, če med njima obstaja pot dolžine dva. V delu predstavimo dosedanje rezultate na področju injektivnih barvanj, osredotočimo pa se na barvanje ravninskih grafov z omejenim notranjim obsegom. Pokažemo, da so ravninski grafi z notranjim obsegom večjim od 18 in maksimalno stopnjo Δ injektivno Δ-obarvljivi, grafi z notranjim obsegom večjim od 9 pa injektivno $(\Delta + 1)$-obarvljivi. Velja tudi, da so $\Delta + 4$ barve dovolj za injektivno obarvanje grafov z notranjim obsegom večjim od 4, če je le Δ zadosti velik. Ti rezultati so zanimivi predvsem zato, ker poznamo ravninske grafe, ki imajo injektivno kromatično število enako $\frac{3}{2}\Delta$.

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Ključne besede:
teorija grafov, barvanje grafov, injektivno barvanje grafov, metoda prenosa naboja

Keywords:
graph theory, graph colorings, injective graph colorings, discharging method

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