

Povzetek

V diplomskem delu je predstavljen aproksimacijski algoritem za NP-težek problem pasovnosti grafa. *Linearna ureditev* grafa na n vozliščih je bijektivna preslikava množice vozlišč na množico $\{1, \dots, n\}$. *Pasovnost linearne ureditve* je največja razlika med preslikanimi vrednostmi sosednjih vozlišč. Najti linearno ureditev z najmanjšo možno pasovnostjo je NP-težek problem. Predstavljen in analiziran je Feigejev verjetnostni algoritem, ki teče v skoraj linearnem času in vrne linearno ureditev, katere pasovnost aproksimira vrednost optimalne pasovnosti znotraj polilogaritemskega faktorja [11]. Algoritem temelji na novem pojmu, imenovanem *široka vložitev*, ki je naravna posplošitev *vložitve majhnih popačenj* Bourgaina in Liniala, Londona ter Rabinovicha.

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Ključne besede: pasovnost, verjetnostni algoritem, aproksimacijski algoritem, široka vložitev.

Key words: bandwidth, randomized algorithm, approximation algorithm, volume respecting embedding.

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