Povzetek

Na začetku opišemo kvadraturne formule, iz katerih izhajajo Gauss-Kronrod-ove kvadraturne formule in predstavimo adaptivne metode.

V drugem poglavju pokažemo, da iz dane tričlenske rekurzivne zveze ortogonalnih polinomov generiranih iz utežne funkcije lahko izpeljemo Gaussovo kvadraturno formulo na n točkah z izračunom delnega spektralnega razcepa tridiagonalne matrike reda n.

V tretjem poglavju opišemo dva algoritma za izračun Gauss-Kronrodove kvadraturne formule na $2n+1$ točkah z realnimi vozli in pozitivnimi utežmi. Oba uporabljata algoritem za izračun Gaussove kvadraturne formule.

V zadnjem poglavju so podani numerični primeri adaptivnih metod.

Ključne besede: numerična integracija, kvadraturne formule, lastne vrednosti in lastni vektorji, ortogonalni polinomi, Gauss-Kronrodove kvadraturne formule

Abstract

We first describe some quadrature rules, predecessors of Gauss-Kronrod quadrature rule and introduce adaptive methods.

In the second chapter it is shown that given the three term recurrence relation for the orthogonal polynomials generated by the weight function, the n-point Gaussian quadrature rule can be generated by computing partial spectral factorization of the symmetric tridiagonal matrix of order n.

In the third chapter we describe two algorithms for the computation of $(2n+1)$-point Gauss-Kronrod quadrature rules with real nodes and positive weights. Both of them use the algorithm for the computation of Gaussian quadrature rule.

Numerical examples of adaptive methods are given in the last chapter.

Key words: numerical integration, quadrature rules, eigenvalues and eigenvectors, orthogonal polynomials, Gauss-Kronrod quadrature rules

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