Povzetek

Če imamo dano približno matriko lastnih vektorjev $\tilde{U}$ za nesingularno hermitsko matriko H, jo lahko diagonaliziramo tako, da najprej izračunamo podobno matriko $H' = \tilde{U}^* H \tilde{U}$ in slednjo diagonaliziramo z Jacobijevo metodo. To delo je posvečeno predvsem numerični stabilnosti pretvorbe matrike H v H' z uporabo aritmetike v plavajoči vejici ter natančnim ocenam relativnih napak lastnih vrednosti. Naša analiza pokaže, da bodo lastne vrednosti izračunane z majhno relativno napako, če so le izračunani približni lastni vektorji dovolj ortonormalni in če je matrika H dovolj pohlevna, torej da majhne relativne motnje elementov matrike povzročijo le majhne relativne spremembe lastnih vrednosti. Pokažemo, da so hermitske matrike pohlevne, kadar je matrika A v razcepu $|H'| = \Delta A \Delta$, $|H'| = \sqrt{(H')^2}$, dovolj dobro pogojena. Kadar $\tilde{U}$ izračunamo s hitro metodo, ki temelji na tridiagonalizaciji, nam da ta postopek predpogojevanja v večini primerov lastne vrednosti izračunane z visoko relativno natančnostjo in porabi manj časa, kot pa Jacobijeva metoda sama.

Ključne besede: simetrični problem lastnih vrednosti, teorija relativnih motenj, Jacobijeva metoda, predpogojevanje, pohlevne matrike.

Matematična klasifikacija: 65F15, 65G05

Abstract

Given approximate eigenvector matrix $\tilde{U}$ of a Hermitian nonsingular matrix H, the spectral decomposition of H can be obtained by computing $H' = \tilde{U}^* H \tilde{U}$ and then diagonalizing H' by Jacobi method. This work addresses the issue of numerical stability of the transition from H to H' in finite precision floating-point arithmetic and rigorous perturbation analysis of relative errors for calculated eigenvalues. Our analysis shows that the eigenvalues will be computed with small relative error if the approximate eigenvectors in $\tilde{U}$ are sufficiently orthonormal and H is well behaved. It means that small relative perturbations in matrix elements cause only small relative changes in eigenvalues. We show that Hermitian matrix is well behaved when A in decomposition $|H'| = \Delta A \Delta$, $|H'| = \sqrt{(H')^2}$, is well-conditioned. If $\tilde{U}$ is computed by fast eigensolver based on tridiagonalization, this preconditioning procedure usually gives the eigensolution with high relative accuracy and it is more efficient than accurate Jacobi type methods on their own.

Key words: symmetric eigenvalue problem, relative perturbation theory, Jacobi method, preconditioning, well behaved matrices

Mathemathics Subject Classifications: 65F15, 65G05

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