

## POVZETEK

Za NP-polne probleme ni znano, ali so rešljivi v polinomskem času. Pač pa so rešljivi v polinomskem času na nedeterminističnem stroju. Med seboj so si po časovni zahtevnosti ekvivalentni: če je le eden med njimi rešljiv v polinomskem času, potem velja to za vse. Še več, potem so rešljivi v polinomskem času sploh vsi problemi, ki so rešljivi v polinomskem času na nedeterminističnem stroju (to je, vsi problemi iz NP). Torej so NP-polni problemi (do polinomske superpozicije natančno) časovno najzahtevnejši med vsemi problemi iz NP. Med NP-polnimi problemi so npr.: ugotoviti, ali je dana sestavljena izjava neprotislovna; poiskati kromatično število danega grafa; ugotoviti, ali ima dani graf Hamiltonov cikel; iz dane končne družine končnih množic izbrati po moči najmanjšo poddružino z isto unijo; ugotoviti, ali se da dano končno množico naravnih števil razdeliti na dva dela z enako vsoto; več problemov<sup>V</sup> zvezi z optimalnim razporejanjem poslov na enem ali več procesorjih; ugotoviti, ali je dana kvadratna diofantska enačba z dvema neznankama rešljiva, itd.

V prvem razdelku sta na kratko opisana stroja z naključnim dostopom in s shranjenim programom ter definiran determ. in nedeterm. Turingov stroj. Navedeni so izreki o ekvivalenci računske moči teh modelov in o polinomski povezanosti časovne zahtevnosti njihovih determ. verzij in modelov, ki bi ustrezali višjim programskim jezikom. Dokazana sta izreka o kvadratni enotračni in eksponentni determ. simulaciji. V drugem razdelku so navedene definicije razredov P in NP, NP-polnosti in prevedljivosti v polinomskem času. Dokazan je izrek o NP-polnosti problema izpolnjivosti. V tretjem razdelku je naštetih nad 60 različnih NP-polnih problemov. Nadalje je navedenih še nekaj NP-težkih problemov in nekaj zanimivih problemov iz NP. V zadnjem razdelku je zbranih nekaj rezultatov v zvezi s približnim reševanjem optimizacijskih problemov.

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