

## Povzetek

Delo obravnava dvojico problemov v superalgebrah.

Začnemo s kratkima dokazoma Frobeniusovega o asociativnih in Zornovega izreka o alternativnih realnih algebrah, ki so hkrati obsegi. Le-ta opredelita prve tri oziroma štiri Cayley-Dicksonove algebre. Potem vpeljemo in študiramo razred realnih enotskih neasociativnih algeber, v katerih je podalgebra, generirana z neskalarним elementom, izomorfna  $\mathbb{C}$ . Imenujemo jih *lokalno kompleksne algebre*. Opišemo vse take algebre do razsežnosti 4. Namen vpeljave omenjenih algeber je predvsem razširitev Frobeniusovega in Zornovega izreka, ki zaobjame še peto Cayley-Dicksonovo algebro, sedenione.

Drugi problem se nanaša na liejeve superavtomorfizme asociativnih prasuperalgeber. Dobimo dokončen odgovor za centralno enostavne superalgebre; njih liejevi superavtomorfizmi so standardnih oblik. Izvzeti je potrebno algebre razsežnosti 2 in 4.

## Abstract

Two problems on superalgebras are examined.

We begin with short proofs of classical theorems by Frobenius and (resp.) Zorn on associative and (resp.) alternative real division algebras. These theorems characterize the first three (resp. four) Cayley-Dickson algebras. Then we introduce and study the class of real unital nonassociative algebras in which the subalgebra generated by any nonscalar element is isomorphic to  $\mathbb{C}$ . We call them *locally complex algebras*. In particular, we describe all such algebras that have dimension at most 4. Our main motivation, however, for introducing locally complex algebras is that this concept makes it possible for us to extend Frobenius' and Zorn's theorems in a way that it also involves the fifth Cayley-Dickson algebra, the sedenions.

The second problem considers Lie superautomorphisms of prime associative superalgebras. A definitive result is obtained for central simple superalgebras: their Lie superautomorphisms are of standard forms, except when the dimension of the superalgebra in question is 2 or 4.

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**Ključne besede:** Cayley-Dicksonove algebre, superalgebra, lokalno kompleksna algebra, sedenioni, liejevi superavtomorfizmi.

**Keywords:** Cayley-Dickson algebras, superalgebra, locally complex algebra, sedenions, lie superautomorphisms.

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