

## Povzetek

To diplomsko delo obravnava matrično monotone funkcije določenega reda iz dveh zornih kotov.

Prvi zapade v matematično analizo; zanima nas kakšno zveznost in nadalje odvedljivost (gladkost) lahko pričakujemo v splošnem od njih. Te lastnosti nas slednjič privedejo do karakterizacij teh funkcij z njihovo gladkostjo in zadoščanjem določenim diferencialnim neenačbam, podanih s pozitivno semidefinitnostjo matrike Taylorjevih koeficientov funkcije.

Drugi zorni kot spada bolj v linearno algebro; gre za netrivialno poostritev pogoja matrične monotonosti, podobno strogi monotonosti za navadne monotone funkcije.

Značilni sta tehniki dokazovanja izrekov. Tam kjer je gladkost funkcij privzeta, nam zadošča poznavanje lastnosti Hadamardovega produkta in njegove povezanosti z diferencialom matrične funkcije na hermitskih matrikah. Sicer pa je potrebno uporabljati tehniko regularizacij, tj. "zglajenja" neustreznih funkcij.

## Abstract

This diploma discusses monotone matrix functions of determined order from two points of view.

First falls in mathematical analysis; we are interested in what continuity and furthermore differentiability (smoothness) can be expected in generally. These properties lead us finally to characterizations of these functions with their smoothness and to satisfying certain differential inequalities, given with positive semidefiniteness of matrix from Taylor's coefficients of function.

Second point of view belongs more to linear algebra; nontrivial intensified condition of matrix monotonicity, similar to strict monotonicity for ordinary monotone functions, is treated.

Techniques used in proofs are characteristic. Where we assume smoothness of functions, knowing properties of Hadamard product and its connection with differential of matrix functions, satisfies. Otherwise we must use regularization technique, to "smooth" unsuitable functions.

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**Key words:** Function: monotone matrix, monotone operator, strongly matrix monotone, monotone increasing, (absolutely) continuous, Lipschitz, (continuously) differentiable, convex. Matrix: Löwner, hermitian, positive (semi)definite, unitary. Determinant: Löwner, extended Löwner; Minor: main, leading. Hadamard product. Daleckii-Krein formula. Regularizations. Theorem of: Rayleigh-Ritz, Schur, Löwner.

# Literatura

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